Differentiation
अवकलन

$C = 2\pi r$

$V = \frac{4\pi}{3} r^3$

Δy Δx

$y = f(x)$

dx dy

y, x पर निर्भर है।

$\frac{dy}{dx}$ = y का अवकलन (differentiation)
x के सापेक्ष (with respect to x)

=

* $V = x^2 + x + 1$

$\frac{dV}{dx}$

ds dt

* $S = t^2 + 1$

$\frac{ds}{dt}$

> नियम

1. $y = x^n$ $n \rightarrow$ पूर्णांक (संख्या)

$$\frac{d}{dx}(x^1) = 1$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^n) = n x^{n-1}$$

$$\frac{d}{dx}(x^{2022}) = 2022 x^{2021}$$

Eg

$$\frac{d}{dx}(x^2) = 2x^{2-1}$$
$$= 2x$$

$$\frac{d}{dx}(x^3) = 3x^2 \quad \frac{d}{dx}(x^4) = 4x^3 \quad \frac{d}{dx}(x^5) = 5x^4$$

$\gg$

1) $\dfrac{d}{dx}\left(\dfrac{1}{x^1}\right) = \dfrac{d}{dx}\left(x^{-1}\right) = -1 \cdot x^{-1-1} = -1 \cdot x^{-2}$ $\quad x^n$

$= -1 \times \dfrac{1}{x^2} = \dfrac{-1}{x^2}$

$\dfrac{d}{dx}\left(\dfrac{1}{x^5}\right) = \dfrac{-5}{x^6}$

2) $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right) = \dfrac{d}{dx}\left(x^{-2}\right) = -2x^{-2-1} = -2x^{-3}$

$= \dfrac{-2}{x^3}$

$\dfrac{d}{dx}\left(\dfrac{1}{x^6}\right) = \dfrac{-6}{x^7}$

3) $\dfrac{d}{dx}\left(\dfrac{1}{x^3}\right) = \dfrac{-3}{x^4}$

$\dfrac{d}{dx}\left(\dfrac{1}{x^7}\right) = \dfrac{-7}{x^8}$

4) $\dfrac{d}{dx}\left(\dfrac{1}{x^4}\right) = \dfrac{-4}{x^5}$ Ans

$\dfrac{d}{dx}\left(\dfrac{1}{x^n}\right) = \dfrac{-n}{x^{n+1}}$

$$\frac{d}{dx}\left(\sqrt{x}\right) = \frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}\cdot x^{-\frac{1}{2}}$$

$$= \frac{1}{2}\cdot\frac{1}{x^{\frac{1}{2}}} = \frac{1}{2}\cdot\frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\left(x^{1}\sqrt{x}\right) = \frac{d}{dx}\left(x^{1}\cdot x^{\frac{1}{2}}\right) = \frac{d}{dx}\left(x^{3/2}\right)$$

$$= \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}$$

$$\boxed{\frac{d}{dx}\left(x^{2}\sqrt{x}\right)} = \frac{d}{dx}\left(x^{2}\cdot x^{\frac{1}{2}}\right) = \frac{d}{dx}\left(x^{5/2}\right)$$

$$= \frac{5}{2}\cdot x^{\frac{5}{2}-1} = \frac{5}{2}\cdot x^{\frac{3}{2}} = \frac{5x\sqrt{x}}{2}$$

$3+\frac{1}{2}$

$\frac{7}{2}$

$$\frac{d}{dx}\left(x^{3}\sqrt{x}\right) = \frac{7}{2}\cdot x^{2}\sqrt{x}$$

$$\frac{d}{dx}\left(x^{4}\sqrt{x}\right) = \frac{9}{2}x^{3}\sqrt{x}$$

$$\frac{d}{dx}\left(x^{5}\sqrt{x}\right) = \frac{11}{2}x^{4}\sqrt{x}$$

$$\frac{d}{dx}\left(x^{6}\sqrt{x}\right) = \frac{13}{2}x^{5}\sqrt{x}$$

$$\frac{d}{dx}\left(x^{n}\sqrt{x}\right) \boxed{= \frac{2n+1}{2}\cdot x^{n-1}\sqrt{x}}$$

Q: $y = x^{19}\sqrt{x}$

$$\frac{dy}{dx} = \boxed{\frac{39}{2}x^{18}\sqrt{x}}$$

Q: $y = x^{21}\sqrt{x}$

$$\frac{dy}{dx} = \boxed{\frac{43}{2}x^{20}\sqrt{x}}$$

1. $S = t^2 + t + 5$

$$\frac{ds}{dt} = \frac{d}{dt}\left[t^2 + t + 5\right]$$

$$= 2t + 1 + 0$$

$$= 2t + 1$$

2. $V = 2t^2 + 5t + 7$

$$\frac{dv}{dt} = 2 \cdot 2t + 5 \times 1 + 0$$

$$= 4t + 5$$

Q. $V = 2\sqrt{x}$

$$\frac{dV}{dx} = \frac{d}{dx}\left(2\sqrt{x}\right)$$

$$= 2 \cdot \frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \cancel{2} \times \frac{1}{\cancel{2}\sqrt{x}}$$

$$= \frac{1}{\sqrt{x}}$$

$P = 4\sqrt{t}$

$$\frac{dP}{dt} = \frac{d}{dt}\left(4\sqrt{t}\right)$$

$$= 4\,\frac{d}{dt}\left(\sqrt{t}\right)$$

$$= \overset{2}{\cancel{4}} \times \frac{1}{\cancel{2}\sqrt{t}}$$

$$= \frac{2}{\sqrt{t}}$$

>> Imp

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\boxed{\ln x = \log_e x}$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

☆ $$\frac{d}{dx}(\sin 2x) = 2\cos 2x$$

$$\frac{d}{dx}(\sin 4x) = 4\cos 4x$$

$$\frac{d}{dx}[\sin(2x+3)] = 2\cos(2x+3)$$

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$

$$\frac{d}{dx}\left(\sin(4x+5)\right) = 4\times\cos(4x+5)$$

$$\frac{d}{dx}\left(\sin(4x^2)\right) = 8x\cos 4x^2$$

$$\frac{d}{dx}\left(\sin(3x^2)\right) = 6x\cos 3x^2$$

$$\frac{d}{dt}\left(\sin(\omega t+\phi)\right) = \omega\cos(\omega t+\phi)$$

$$\frac{d}{dt}\left(\cos(\omega t+\phi)\right) = -\omega\sin(\omega t+\phi)$$

$$y = \sqrt{x+1} \qquad y = \sqrt{X}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}} \times (1+0)$$

$$= \frac{1}{2\sqrt{x+1}}$$

Eg:

$$y = \sqrt{2x+3}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{2x+3}} \times (2+0) = \frac{\cancel{2}}{\cancel{2}\sqrt{2x+3}} = \frac{1}{\sqrt{2x+3}}$$

$$y = \sqrt{4x+5}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4x+5}} \times (4+0)$$

$$= \frac{\cancel{4}^{2}}{\cancel{2}\sqrt{4x+5}} = \frac{2}{\sqrt{4x+5}}$$ Ans

$$y = \sqrt{x^2 + 1}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x^2+1}} \times (2x + 0)$$

$$= \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$$