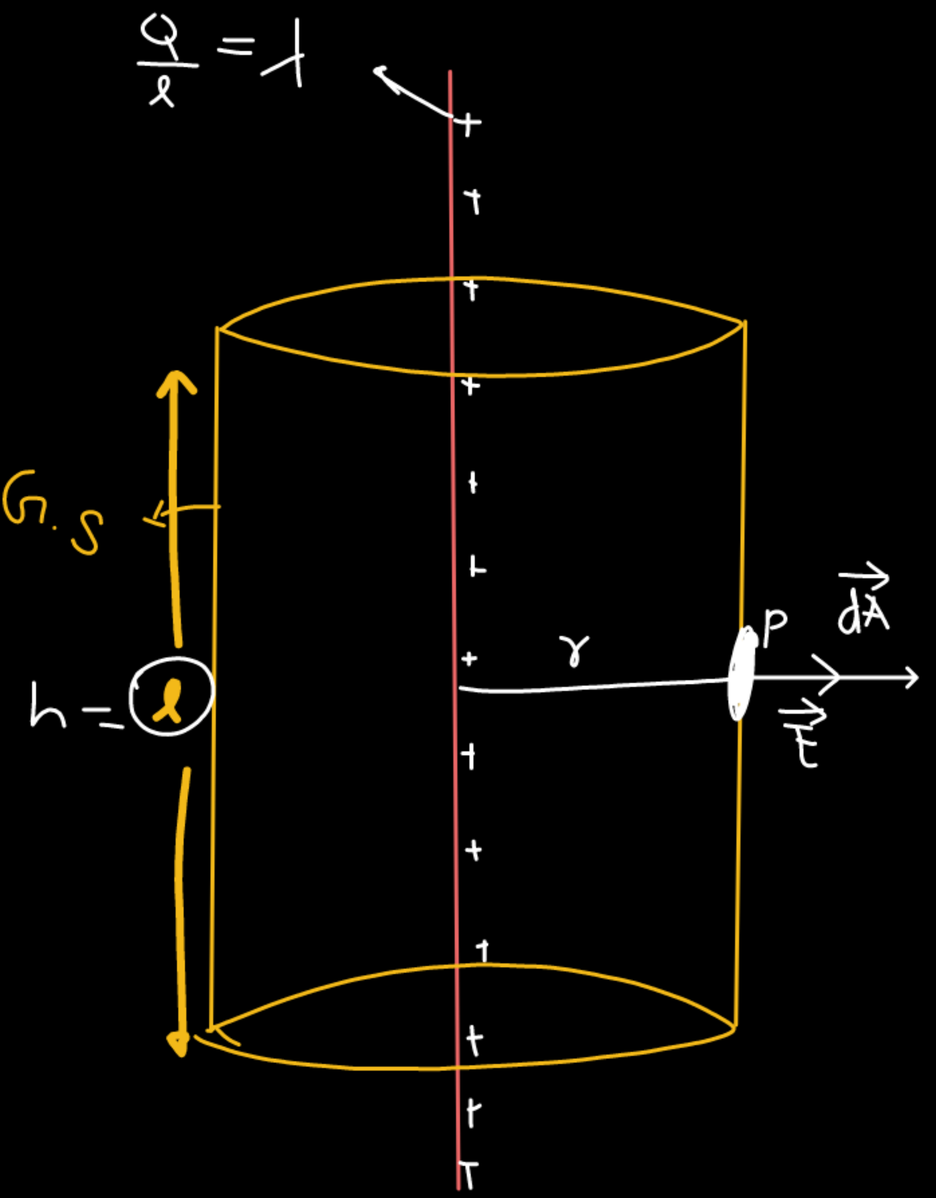
गॉस प्रमेय का अनुप्रयोग

(1) अनन्त लम्बाई वाले आवेशित सीधी चालक तार के कारण विद्युत क्षेत्र

(Electric field due to infinite Conducting Straight wire)



$$Q_{enclosed} = \lambda l$$

गॉस प्रमेय से :

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

$$\oint E \cdot dA \cdot \cos 0^\circ = \frac{1l}{\epsilon_0}$$

$$E \oint dA = \frac{1l}{\epsilon_0}$$

$$E \int dA = \frac{1l}{\epsilon_0}$$

$$EA = \frac{1l}{\epsilon_0}$$

$$E \times 2\pi r \cancel{l} = \frac{1\cancel{l}}{\epsilon_0}$$

$$E \cdot 2\pi r = \frac{1}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0 r}$$

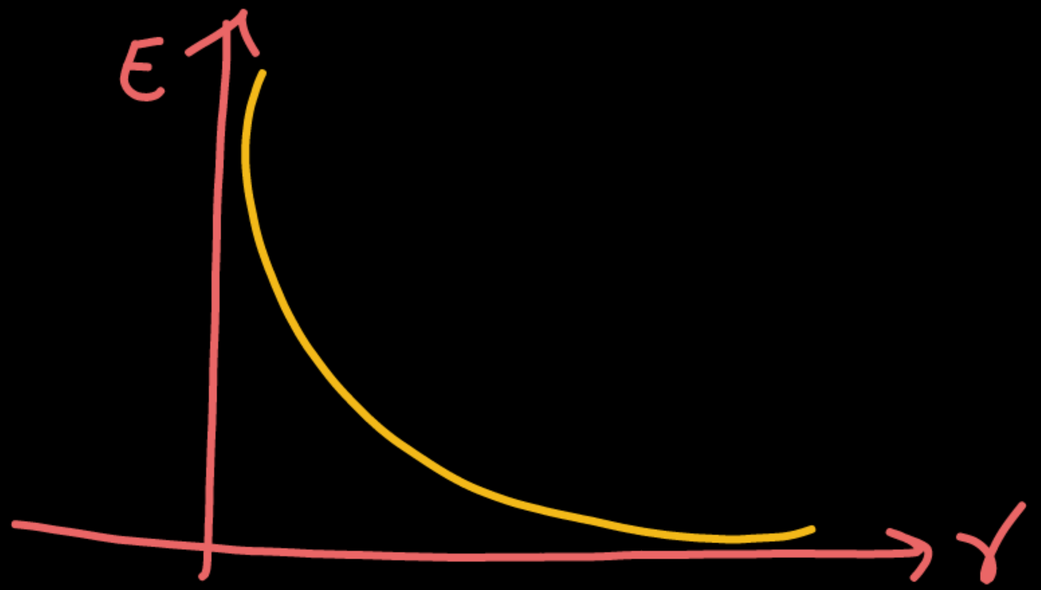
$$E = \frac{2\lambda}{4\pi\epsilon_0 \cdot r} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{2\lambda}{r}\right)$$

$$= \frac{2|k|\lambda}{r}$$

$$\Rightarrow E = \boxed{\frac{\lambda}{2\pi\epsilon_0 r}}$$

Case(I) : अगर λ = नियत हो

$$\boxed{E \propto \frac{1}{r}}$$



Case(II) : r नियत हो

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\boxed{E \propto \lambda}$$

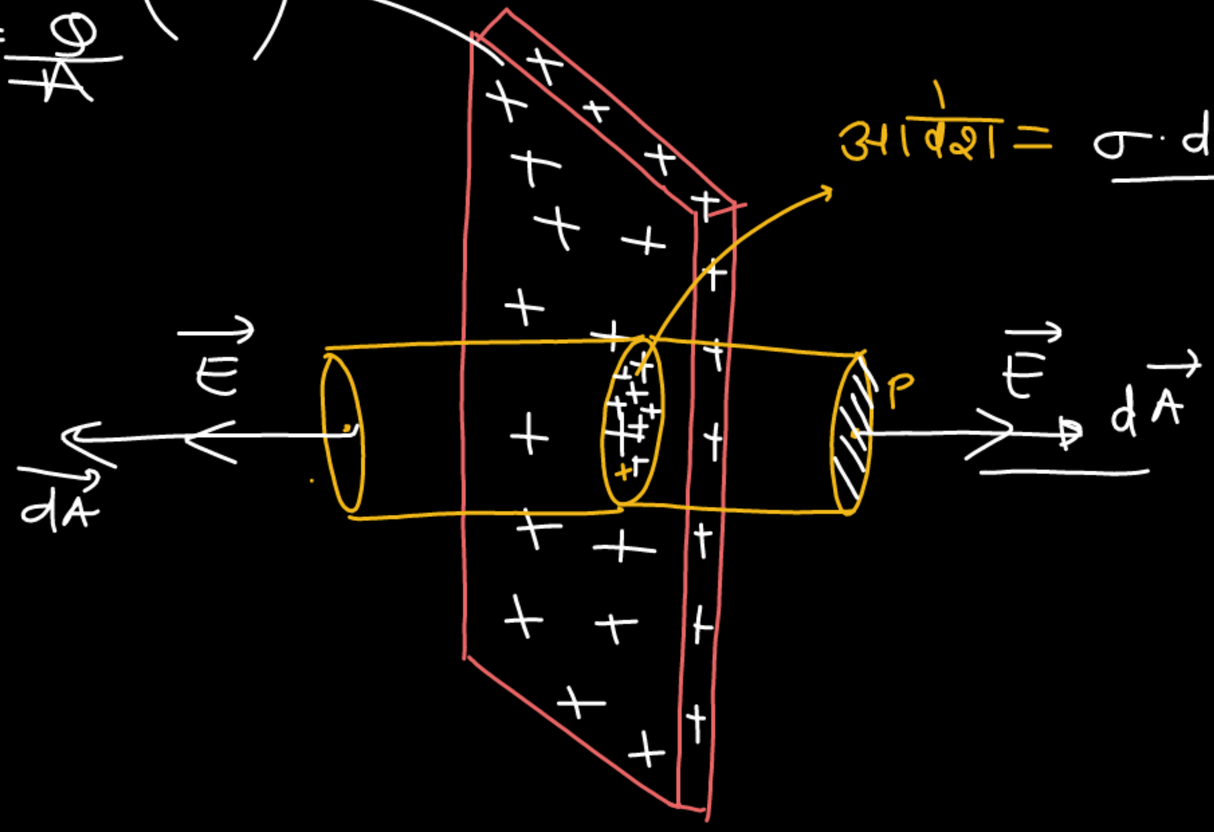
रेखीय आवेश घनत्व

$$\lambda = q/l \rightarrow C/m$$

(2) Electric field due to infinite Plane sheet

अनन्त आवेशित सतह के कारण विद्युत क्षेत्र

पृष्ठीय आवेश घनत्व $= \frac{q}{A}$ (σ)



$$\int \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$E dA \cos 0^\circ + E dA \cos 0^\circ = \frac{\sigma dA}{\epsilon_0}$$

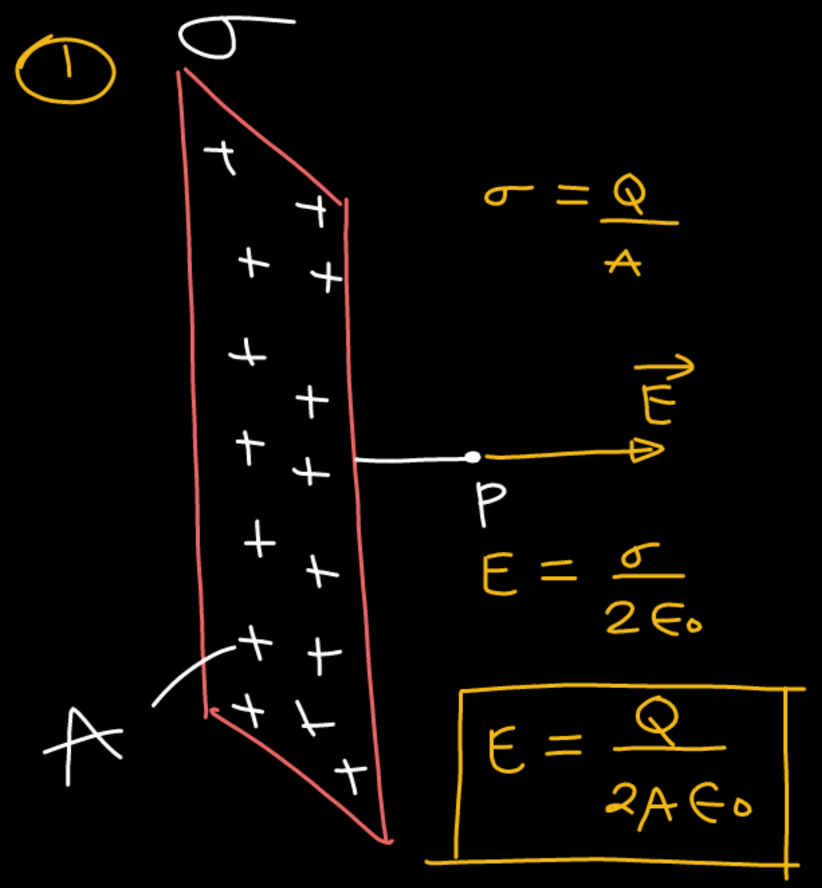
$$E dA + E dA = \frac{\sigma dA}{\epsilon_0}$$

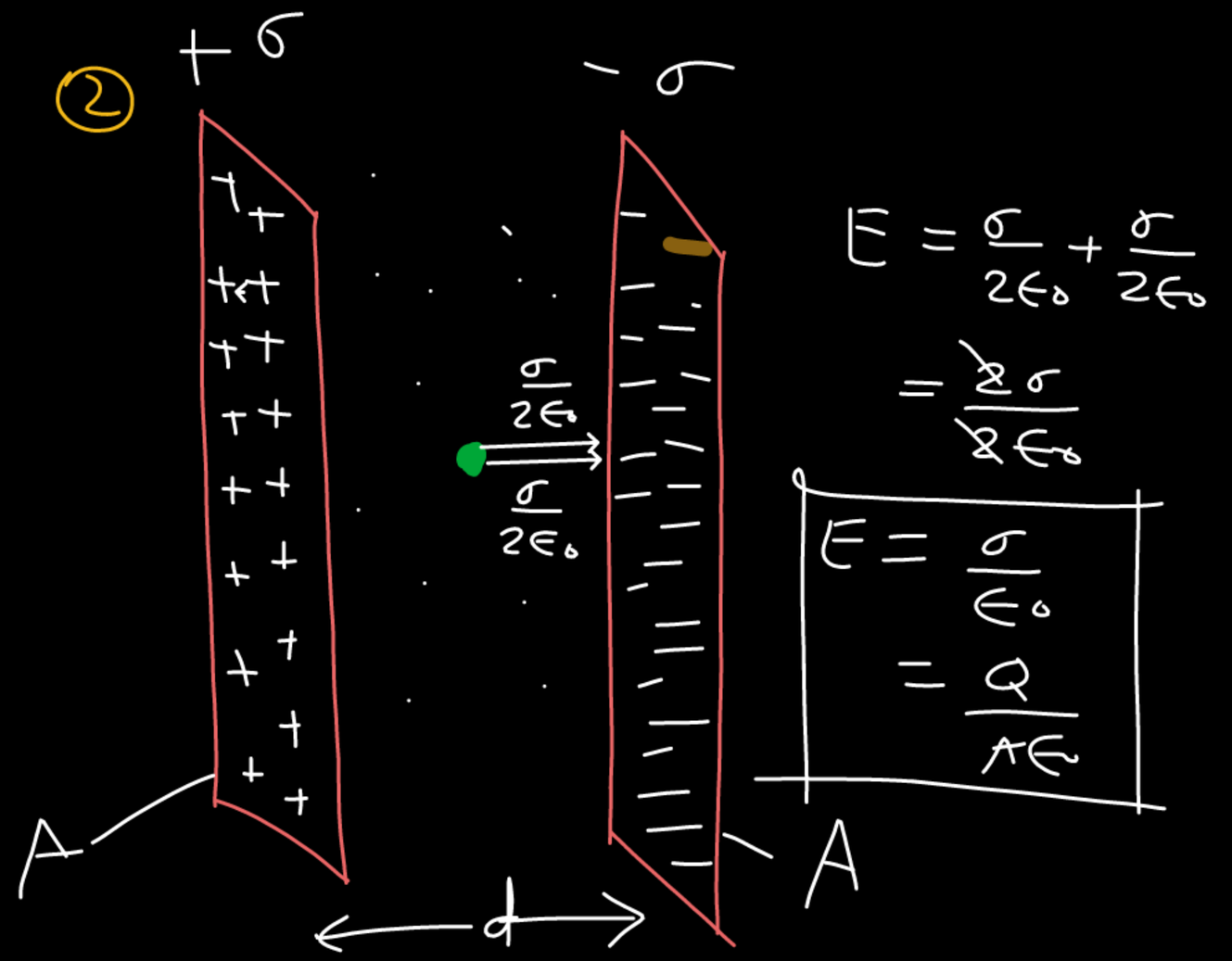
$$2E dA = \frac{\sigma dA}{\epsilon_0}$$

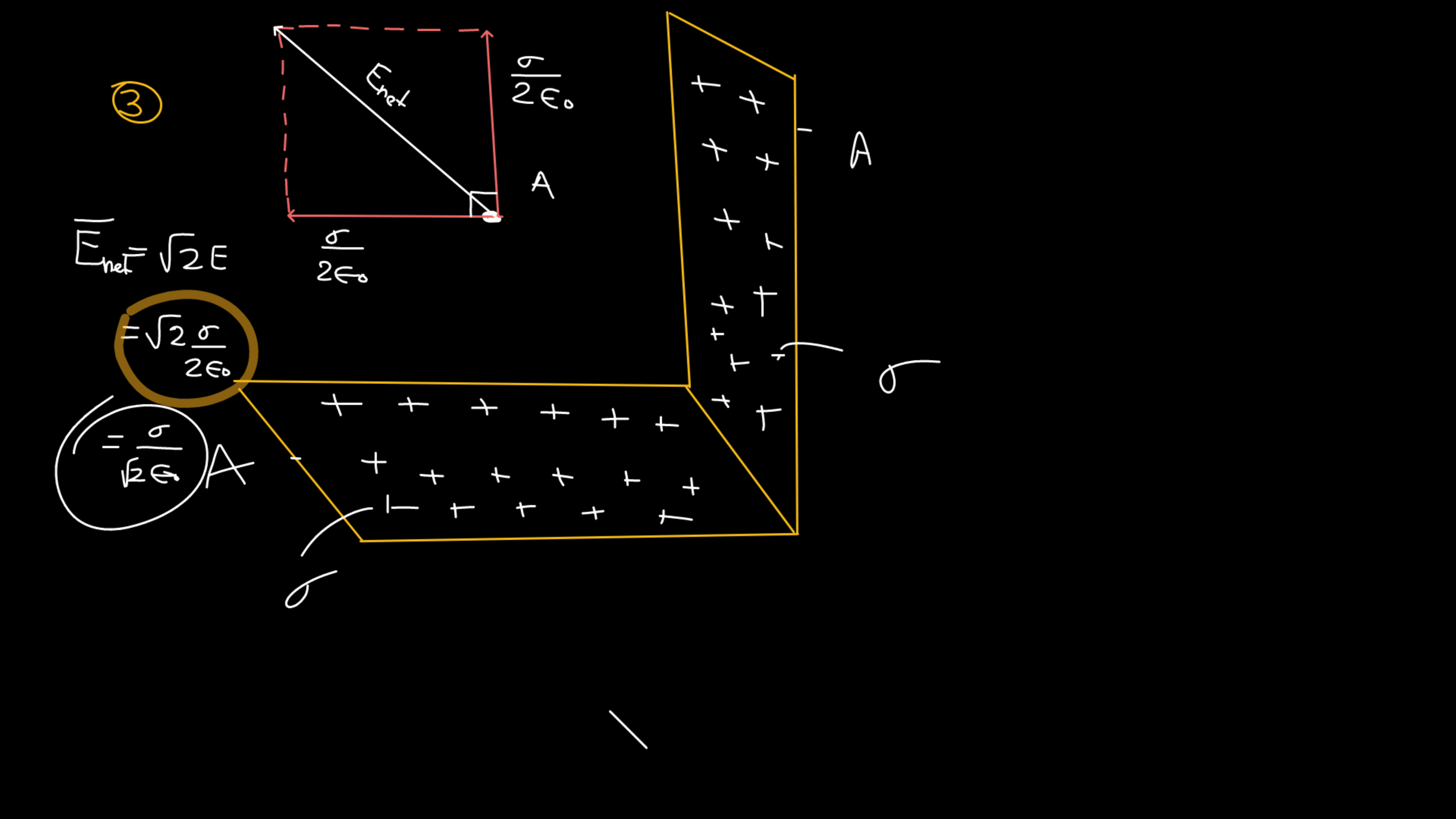
$$2E = \frac{\sigma}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

Example :

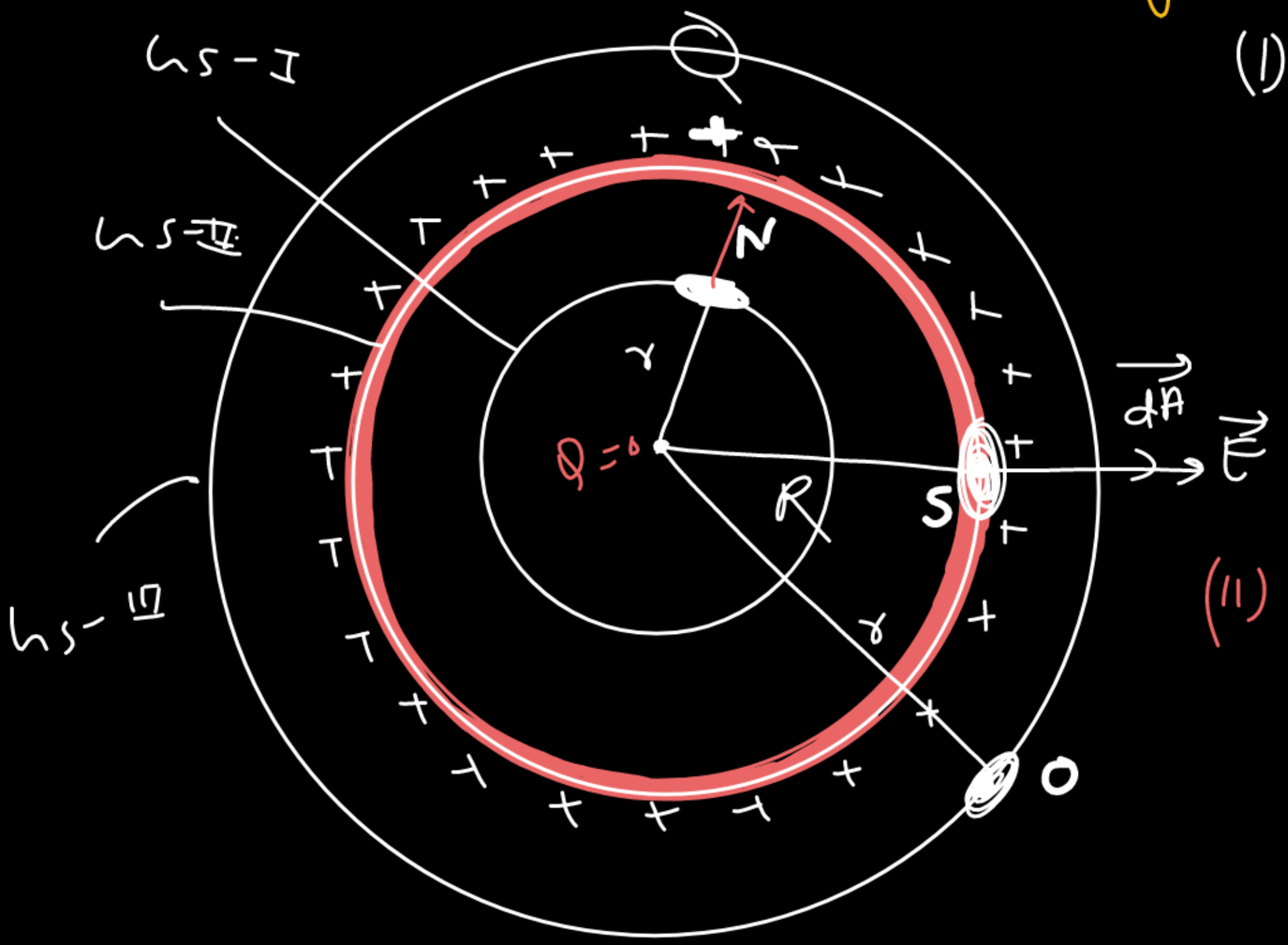

①
σ
$\sigma = \frac{Q}{A}$
E
P
$E = \frac{\sigma}{2\epsilon_0}$
A
$E = \frac{Q}{2A\epsilon_0}$


②
+σ
−σ
$\frac{\sigma}{2\epsilon_0}$
$\frac{\sigma}{2\epsilon_0}$
A
d
$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$
$= \frac{2\sigma}{2\epsilon_0}$
$E = \frac{\sigma}{\epsilon_0}$
$= \frac{Q}{A\epsilon}$
A


③
E_{net}
$\frac{\sigma}{2\epsilon_0}$
A
$\frac{\sigma}{2\epsilon_0}$
$E_{net} = \sqrt{2}E$
$= \sqrt{2}\frac{\sigma}{2\epsilon_0}$
$= \frac{\sigma}{\sqrt{2}\epsilon_0}A$
A
σ
σ

③ आवेशित खोखला गोला के कारण विद्युत क्षेत्र

Electric field due to Conducting shell | Hollow Sphere.



(I) प्रथम गॉसीय सतह

$r < R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0} = \frac{0}{\epsilon_0}$$

$$\boxed{E = 0}$$

(II) द्वितीय गा. सतह : (R)

$$\oint E\, dA \cos 0^\circ = Q$$

$$E = \frac{Q}{4\pi\epsilon_0 R^2} = \boxed{\frac{kQ}{R}}_{\max}$$

गा-3 :

$r > R$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$\oint E\, dA \cos 0^\circ = \frac{q}{\epsilon_0}$$

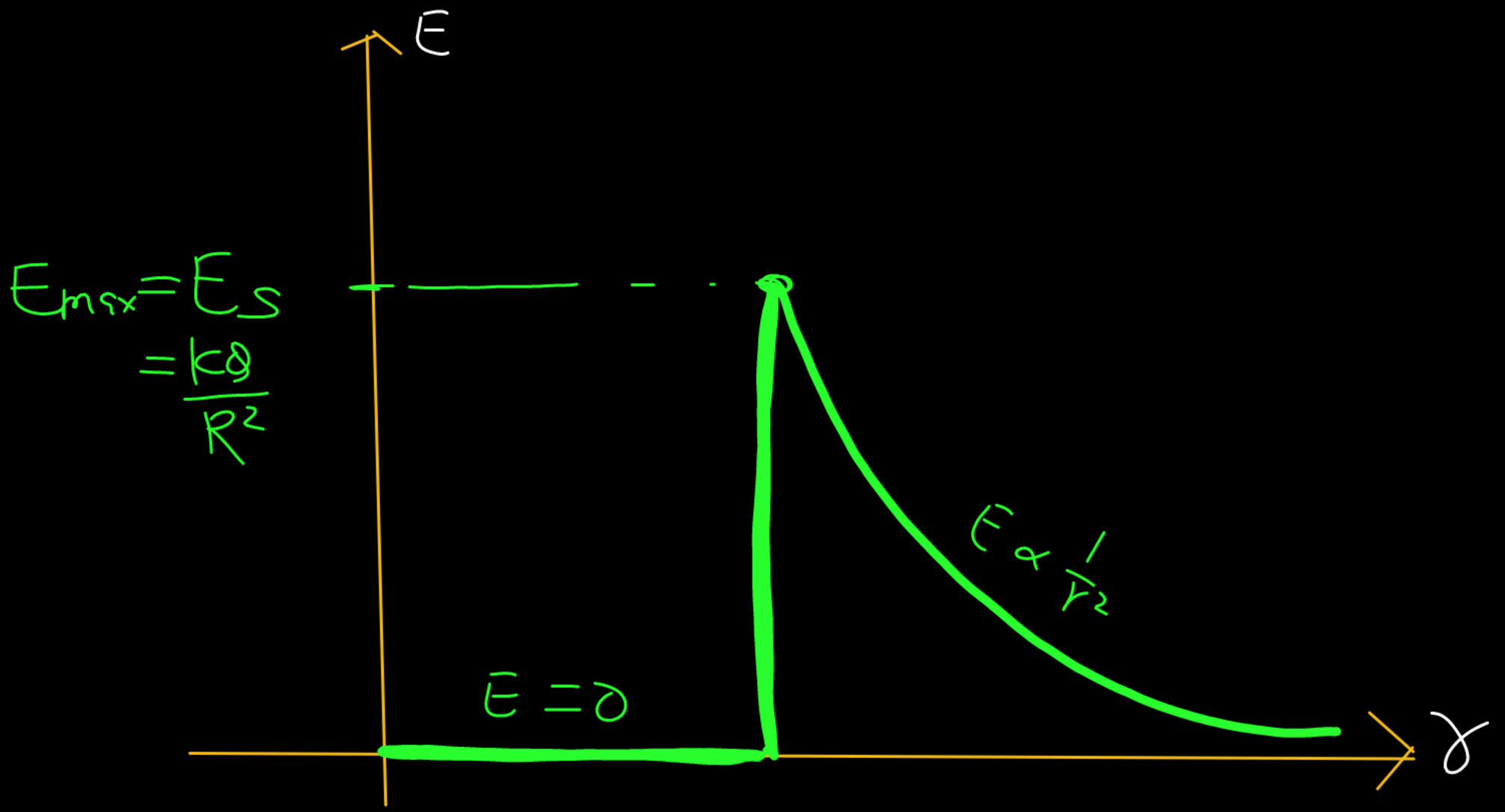
$$E \oint dA = \frac{q}{\epsilon_0}$$

$$E \times A = \frac{q}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$E_{out} = \frac{kQ}{r^2}$$


E
$E_{max} = E_S = \frac{kQ}{R^2}$
$E = 0$
$E \propto \frac{1}{r^2}$
r